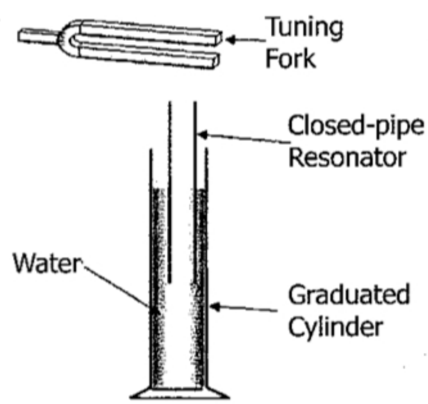
Resonance in Air Columns

When discussing waves in one dimension, we observed that a **standing wave** forms on a spring when reflected waves interfere with incident waves. We learned that the frequencies at which standing waves occur are known as **harmonic frequencies**. Each harmonic frequency is a whole-number multiple of the lowest harmonic frequency, which is called the **fundamental frequency**.

Sound waves are normally three-dimensional. However, when a sound wave travels in a long, narrow column of air (such as in a musical instrument), they become effectively one-dimensional. Under such conditions, it is possible for sound to form a **standing wave**. When a standing wave is formed, the air column will **resonate**. In other words, the amplitude (loudness) of the sound will increase dramatically.

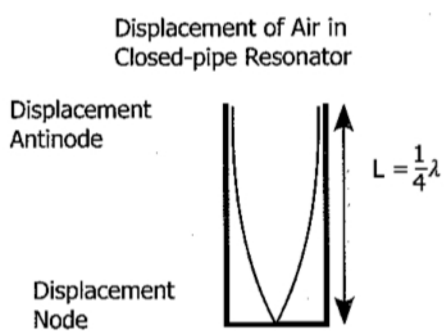
Closed-Pipe Column

A closed-pipe air column is open at one end and closed at the other. The picture on the right illustrates a simple example. To produce resonance, the tuning fork is held near the mouth of the tube. The tube is then raised or lowered until the closed-pipe column resonates. A resonating air column will dramatically increase the volume of the tuning fork.



The shortest closed tube in which resonance can occur is called the **first resonant length**. The frequency of the sound that resonates at the first resonant length is called the **fundamental frequency**.

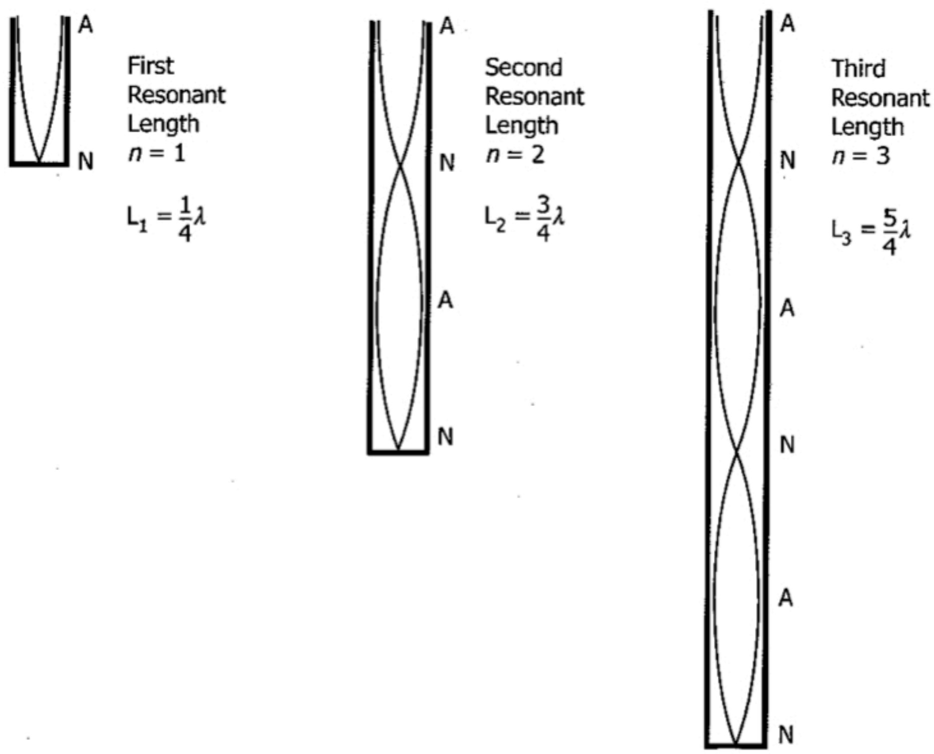
In the case of a closed-pipe resonator, resonance will occur whenever there is a node at the closed end and an antinode at the open end. The diagram below illustrates the first time that this occurs in a closed-pipe.



Notice that the length of the pipe in this case is one-quarter of the wavelength of the sound. Thus, in the case of a closed-pipe resonator, the first resonant length occurs at a length of one-quarter of the wavelength of the sound.

The diagram below illustrates the first few resonant lengths for a closed-pipe resonator.

Resonant Lengths of Closed-Pipe Columns of Air Showing the Displacement of Nodes and Antinodes



From this diagram, it is evident that the resonant lengths of a closed-pipe resonator are odd multiples of one-quarter wavelength.

$$L_n = (2n-1)\frac{\lambda}{4} \qquad \text{(closed-pipe)}$$

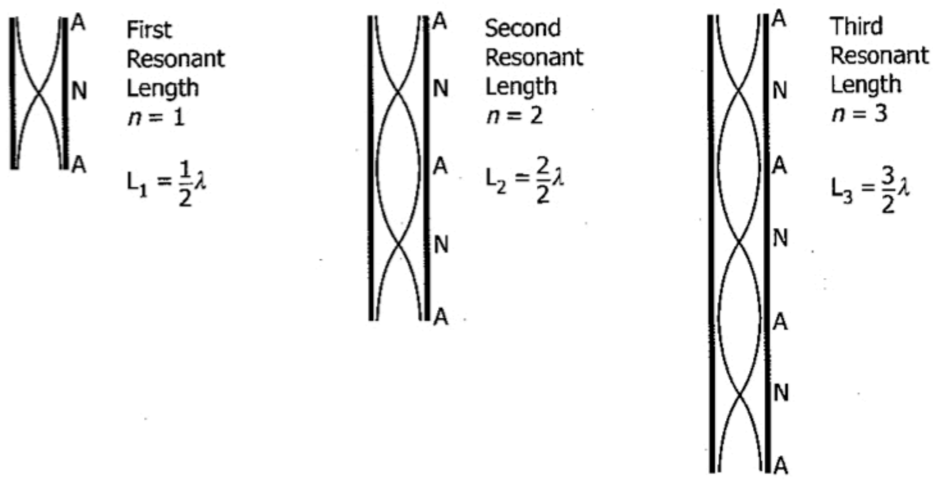
In other words, resonance will occur in a closed-pipe of length $\frac{\lambda}{4}$, $\frac{3\lambda}{4}$, $\frac{5\lambda}{4}$, etc.

Open-Pipe Column

An open-pipe air column is open at both ends.

In the case of an open-pipe resonator, resonance will occur whenever there is an antinode at both open ends. The diagram below illustrates the first few resonant lengths for a closed-pipe resonator.

Resonant Lengths of Open-Pipe Columns of Air Showing the Displacement Nodes and Antinodes



From this diagram, it is evident that the resonant lengths of an open-pipe resonator are multiples of one-half wavelength.

$$L_n = \frac{n\lambda}{2} \qquad \text{(open-pipe)}$$

In other words, resonance will occur in a closed-pipe of length $\frac{\lambda}{2}$, λ, $\frac{3\lambda}{2}$, 2λ, etc.

Example

A vibrating tuning fork is held near the mouth of a closed pipe resonator. The water level in the pipe is lowered, and the first loud sound is heard when the air column is $9.0\ cm$ long. The temperature in the room is $20°\ C$.

a) Calculate the wavelength of the sound produced by the tuning fork.

b) Calculate the length of the air column for the second and third resonances.

c) Estimate the frequency of the tuning fork.

Waves Worksheet #10

1. A tuning fork is held over the open end of a closed air column. The water level in the column is lowered. An increase in loudness is detected when the air column is 17 cm long and again when it is 51 cm long.
   a) Determine the wavelength of the sound produced by the tuning fork.
   b) If the water level continues to be lowered, at what level will the next increase in loudness be heard?

2. The first resonance length of an air column, resonating to a fixed frequency, is 32 cm.
   a) Determine the second and third resonance lengths, if the column is closed at one end.
   b) Determine the second and third resonance lengths, if the column is open at both ends.

3. The third resonance length of a closed air column, resonating to a tuning fork, is 95 cm. Determine the first and second resonance lengths.

4. The second resonance length of an air column, open at both ends and resonating to a fixed frequency, is 64 cm. Determine the first and third resonance lengths.

5. The frequency of a tuning fork is unknown. A student uses an air column at $27°\,C$ and finds resonances spaced by 20.2 cm. What is the frequency of the tuning fork?

6. A particular organ pipe, open at both ends, needs to resonate in its fundamental mode with a frequency of 128 Hz. The organ has been designed to be played at a temperature of $22°\,C$.
   a) How long does the organ pipe need to be?
   b) If this pipe is closed at one end by a stopper, at what fundamental frequency will it resonate?

7. The 440 Hz tuning fork is used with a resonating column to determine the velocity of sound in helium gas. If the spacing between resonances is 110 cm, what is the velocity of sound in helium?